Magnetic monopoles

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1ST
Predicting Magnetic Monopoles
Quantization of charge, gauge symmetries and super massive monopoles

2ND
Looking for magnetic monopoles
Detection experiments, astronomical bounds and the "monopole problem"

3RD
Creating a magnetic monopole
MoEDAL experiment

1ST

Predicting Magnetic Monopoles

Maxwell's equation symmetry
(in gaussian units)

In vaccum

$$\begin{aligned}
\vec{\nabla} \cdot \vec{B} &= 0 \\
\vec{\nabla} \times \vec{E} + \frac{1}{c}\frac{\partial B}{\partial t} &= 0 \\
\vec{\nabla} \cdot \vec{E} &= 0 \\
\vec{\nabla} \times \vec{B} - \frac{1}{c}\frac{\partial E}{\partial t} &= 0
\end{aligned}$$

With charge

$$\begin{aligned}
\vec{\nabla} \cdot \vec{B} &= 0 \\
\vec{\nabla} \times \vec{E} + \frac{1}{c}\frac{\partial B}{\partial t} &= 0 \\
\vec{\nabla} \cdot \vec{E} &= 4\pi\rho \\
\vec{\nabla} \times \vec{B} - \frac{1}{c}\frac{\partial E}{\partial t} &= \frac{4\pi}{c}\vec{J}.
\end{aligned}$$

Maxwell's equation symmetry
(in gaussian units)

In vaccum

$$\begin{aligned}
\vec{\nabla}\cdot\vec{B} &= 0 \\
\vec{\nabla}\times\vec{E}+\frac{1}{c}\frac{\partial B}{\partial t} &= 0 \\
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\vec{\nabla}\times\vec{B}-\frac{1}{c}\frac{\partial E}{\partial t} &= 0
\end{aligned}$$

With charge

$$\begin{aligned}
\vec{\nabla}\cdot\vec{B} &= 4\pi\rho_m \\
\vec{\nabla}\times\vec{E}+\frac{1}{c}\frac{\partial B}{\partial t} &= -\frac{4\pi}{c}\vec{J}_m \\
\vec{\nabla}\cdot\vec{E} &= 4\pi\rho \\
\vec{\nabla}\times\vec{B}-\frac{1}{c}\frac{\partial E}{\partial t} &= \frac{4\pi}{c}\vec{J}.
\end{aligned}$$

The absence of magnetic monopoles

Electrodynamics

$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$\nabla \cdot \vec{B} = \nabla \cdot (\vec{\nabla} \times \vec{A})$$

$$\nabla \cdot \vec{B} = 0$$

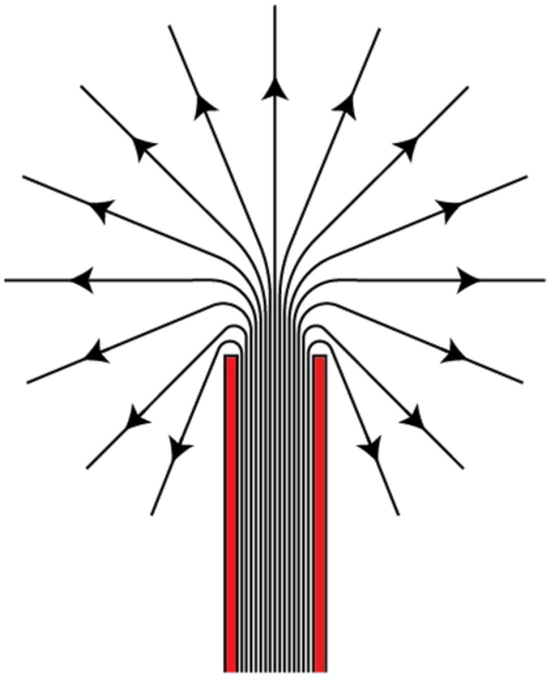
Quantum mechanics

$$|\psi_C >= exp(\frac{-iq}{\hbar c} \int \vec{A} \cdot d\vec{l})|\psi_A >$$

Potential fields forbid magnetic monopoles in ED and QM

Dirac’s argument (1931)

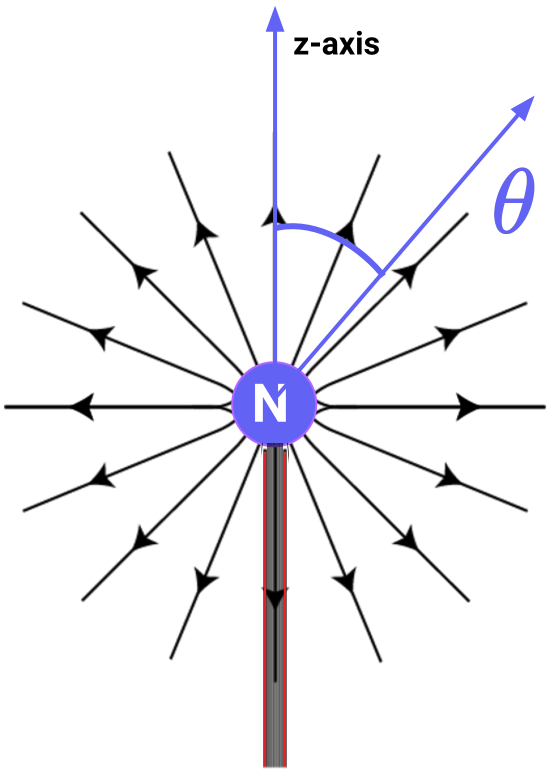
- A current through a solenoid produces magnetic field
- If the solenoid is infinitely long and thin, the field lines never close
- We have something similar to a magnetic monopole, but does the theory hold up?



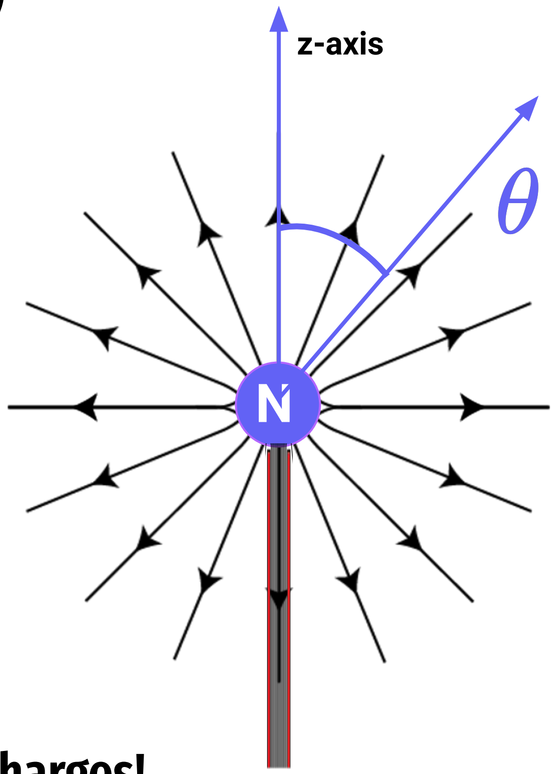
Dirac's argument (1931)

Magnetic field produced: $$\vec{B} = \frac{g}{r^2}\hat{r}$$

A possible vector potential: $$\vec{A} = g\frac{1-cos(\theta)}{rsin(\theta)}\hat{\varphi}$$



Dirac's argument (1931)



Flux of magnetic field:

$$\Phi_B = \int_A \vec{B} \cdot d\vec{S} = \int_\Gamma \vec{A} \cdot d\vec{l}$$
$$= 2\pi g(1 - cos(\theta))$$

$$\Phi_B(\theta = 2\pi) = 4\pi g$$

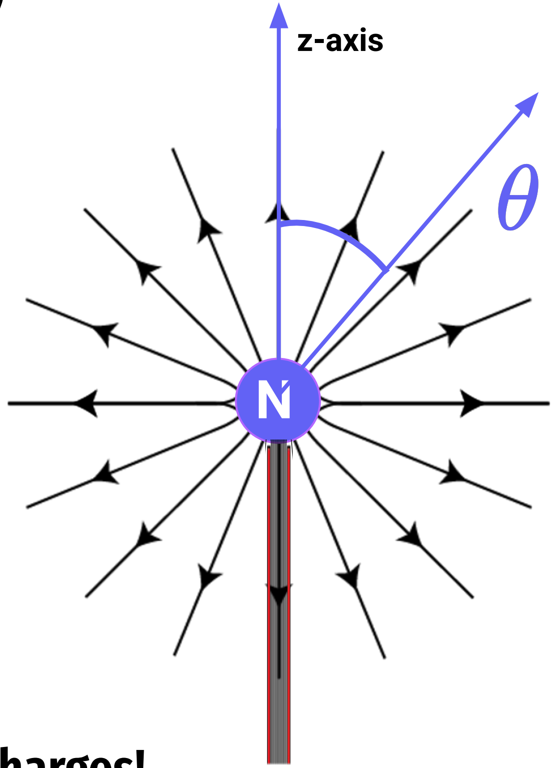
Similar to the gauss law for electric charges!

$$\Phi_E(\theta = 2\pi) = 4\pi q$$

Dirac's argument (1931)

Flux of magnetic field:

$$\Phi_B = \int_A \vec{B} \cdot d\vec{S} = \int_\Gamma \vec{A} \cdot \color{red}{d\vec{l}}$$
$$= 2\pi g(1 - cos(\theta))$$
$$\Phi_B(\theta = 2\pi) = \color{red}{4\pi g}$$



Similar to the gauss law for electric charges!

But is mathematically wrong

Dirac's argument (1931)

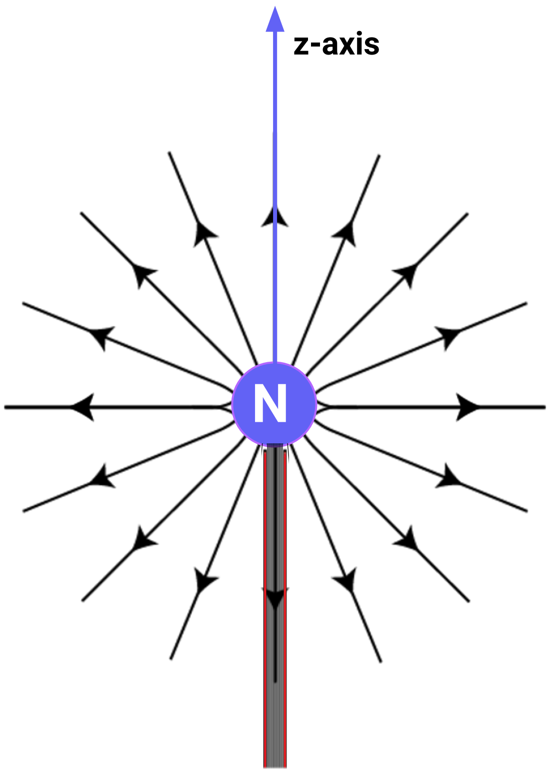
Flux of magnetic field:

$$\Phi_B = \begin{cases} 2\pi g(1 - cos(\theta)); \theta < \pi \\ 0 \quad ; \theta = \pi \end{cases}$$

$$\vec{B} = \nabla \times \vec{A} - 4\pi g \Theta(-z)\delta(x)\delta(y)\hat{z}$$

Dirac string singularity

Physical or mathematical?



Dirac's argument (1931)

Is the dirac string detectable?

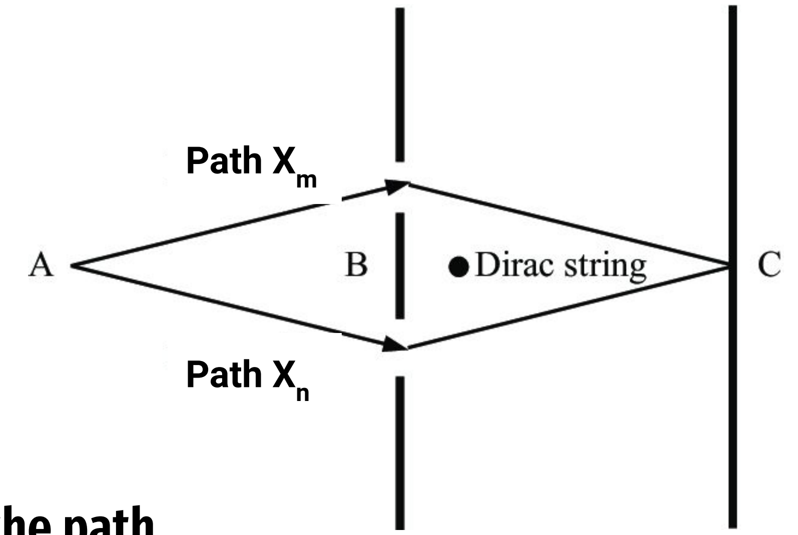
By the Aharonov-bohm effect:

$$|\psi_C >= exp(\frac{-iq}{\hbar c}\int \vec{A}\cdot d\vec{l})|\psi_A >$$

The phase change will be

$$\theta = \frac{q}{\hbar c}\int \vec{A}\cdot d\vec{l}$$

Measuring the phase in C allows one to know the path.



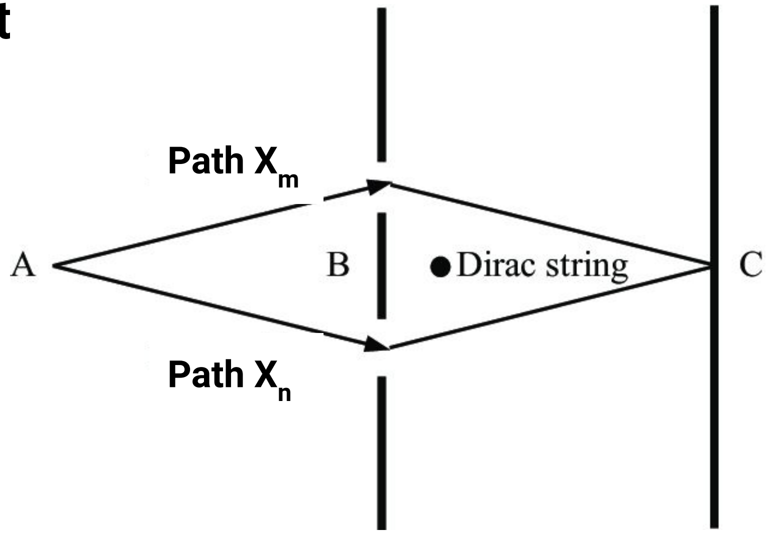
Dirac's argument (1931)

It's possible to calculate the difference in phase shift Between paths.

$$\Delta\theta = \frac{-q}{\hbar c}\int_{A\to X_m\to C}\vec{A}\cdot d\vec{l} - \frac{-q}{\hbar c}\int_{A\to X_n\to C}\vec{A}\cdot d\vec{l}$$

$$\Delta\theta = \frac{-q}{\hbar c}\int_{A}\vec{B}\cdot d\vec{S}$$

$$\boxed{\Delta\theta = \frac{-q}{\hbar c}4\pi g}$$



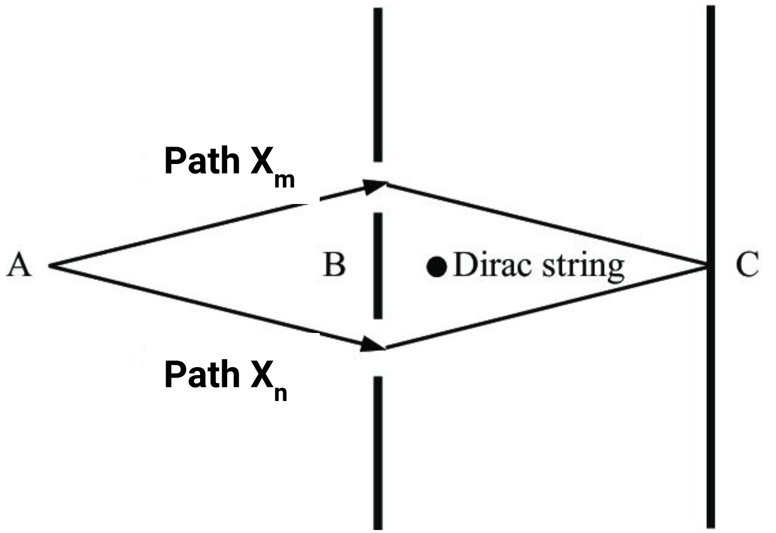
Dirac's argument (1931)

There is a condition where is undetectable:

$$\frac{-q}{\hbar c} 4\pi g = n 2\pi$$

$$\frac{-qg}{\hbar c} \in \mathbf{Z}$$

If charge is quantized, then magnetic monopoles are allowed!

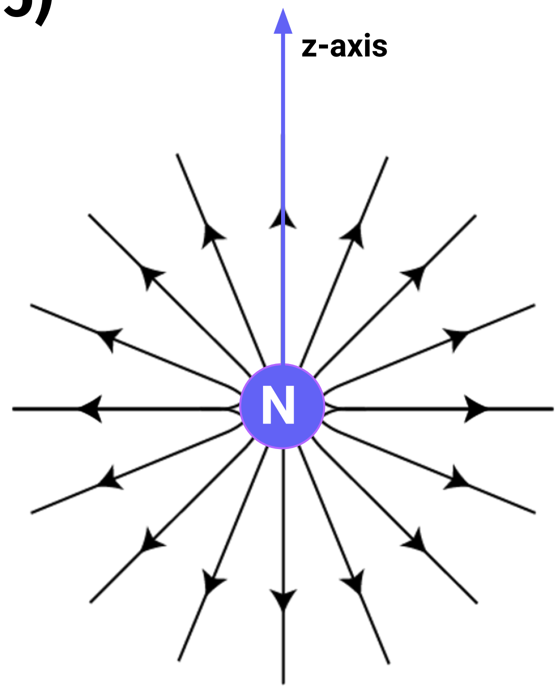


Removing the string (1975)

Charge quantization comes from the dirac string situation. Can we remove the string while getting the quantization condition?

$$\nabla \cdot \vec{B} = \nabla \cdot (\vec{\nabla} \times \vec{A})$$

Not using a vector potential...Let's try to use more



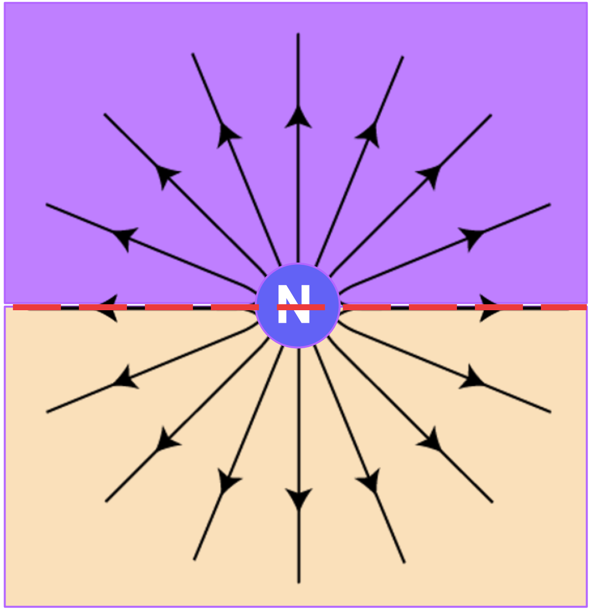
Removing the string (1975)

Let's assume two vector potential in two different regions.

$$\vec{A}_{sup} = g\frac{1-\cos(\theta)}{r\sin(\theta)}\hat{\varphi};\, 0 \leq \theta \leq \frac{\pi}{2}$$

$$\vec{A}_{inf} = -g\frac{1+\cos(\theta)}{r\sin(\theta)}\hat{\varphi};\, \frac{\pi}{2} \leq \theta \leq \pi$$

$$\left.\right\} \vec{B} = \frac{g}{r^2}\hat{r}$$



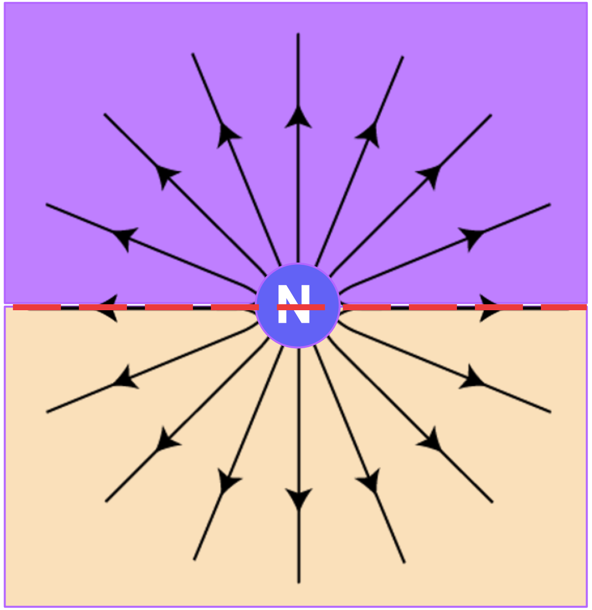
Removing the string (1975)

In the equator, is mandatory that:

$$\vec{\nabla}\chi = \vec{A}_{sup} - \vec{A}_{inf}$$

$$\vec{\nabla}\chi = -\frac{2g}{rsin(\theta)}\hat{\varphi}$$

$$\chi = 2g\varphi$$



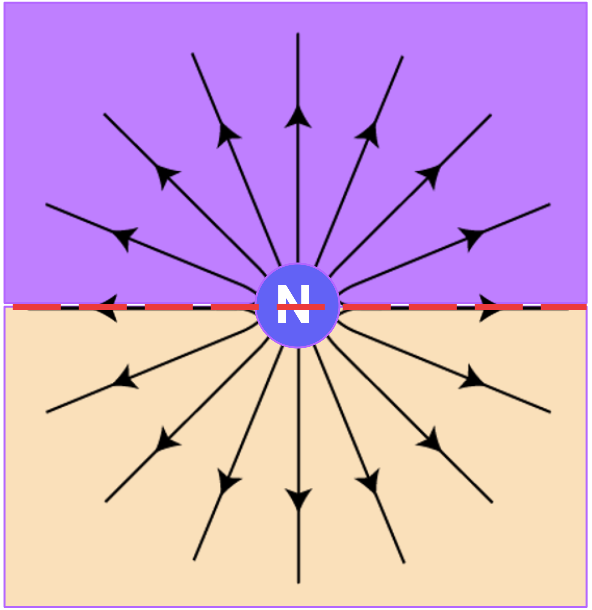
Removing the string (1975)

The same gauge field must be valid in QM:

$$U_g = exp(\frac{-iq\chi(\varphi{=}0)}{\hbar c}) = exp(\frac{-iq\chi(\varphi{=}2\pi)}{\hbar c})$$

$$U_g = exp(0) = exp(\frac{-i4qg\pi}{\hbar c})$$

$$\frac{2qg}{\hbar c} \in \mathbf{Z}$$ The charge quantization is a consequence of gauge symmetries!

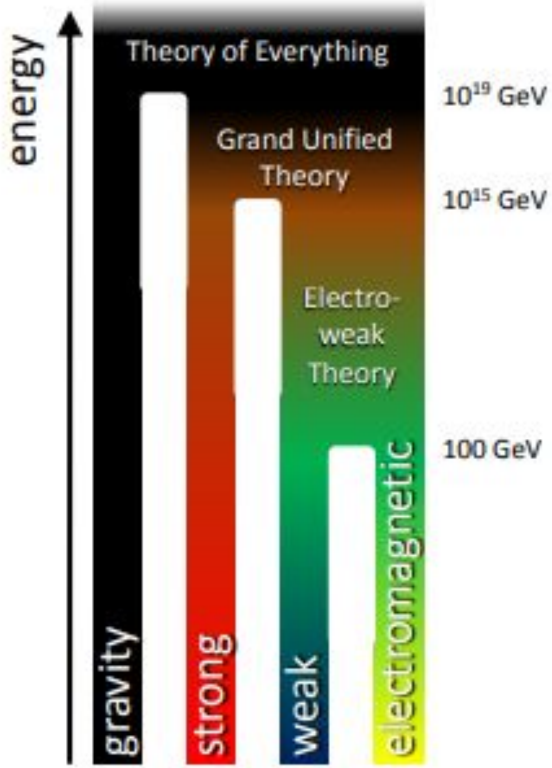


‘T Hooft-Polyakov Monopole(1975)

In simple terms

- The gauge field theory allows one to study the fundamental forces in a unified way in high energies.

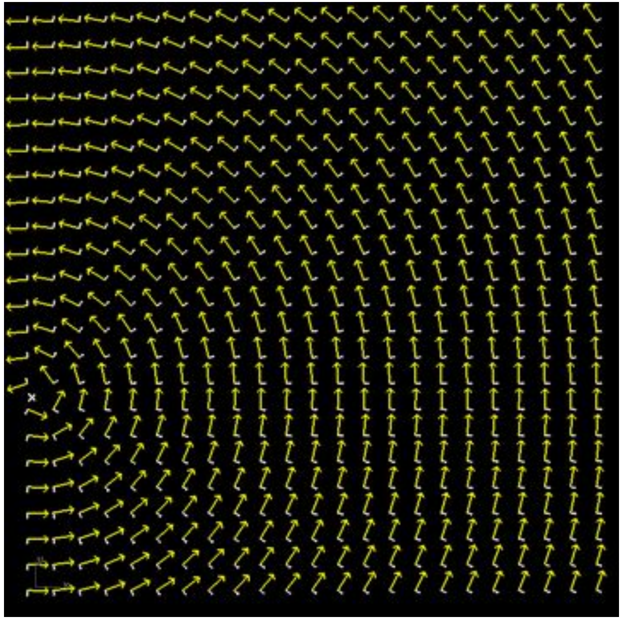
- What separates the two symmetries, at lower energies, is the higgs field.



‘T Hooft-Polyakov Monopole(1975)

In simple terms

- The higgs field in Grand Unified Theories can be *represented as a vector field* with two characteristics.
  - Continuous and invariant under smooth changes (similar to vector field gauge invariance)
  - Non-zero everywhere in vacuum



'T Hooft-Polyakov Monopole(1975)

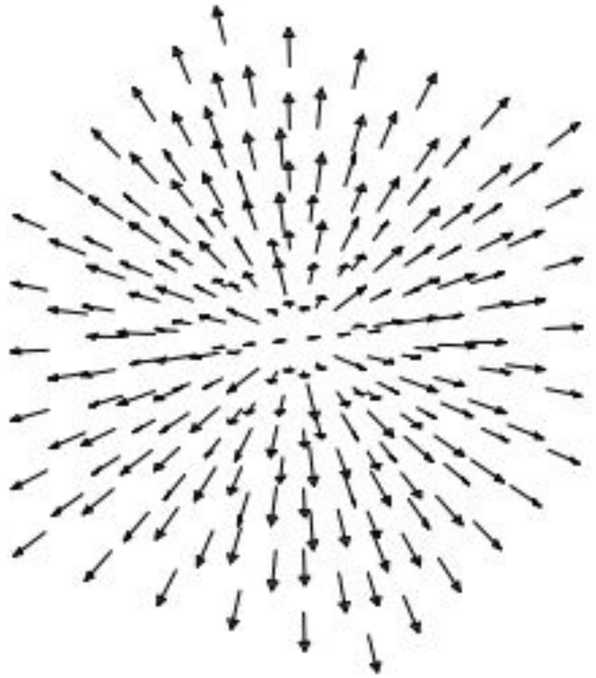
In simple terms

- The "Hedgehog" configuration is a interesting situation.

- Field is zero in the center, which <u>can't be removed by smooth changes.</u>
    - It cannot be vacuum: there is a massive particle in the center ($E \sim 10^{15}$GeV)

- In this situation, the magnetic field can be determined as $$\vec{B} = \frac{g}{r^2}\hat{r}$$

- Gauge theories inevitably predict magnetic monopoles!



2ST

Looking for magnetic monopoles

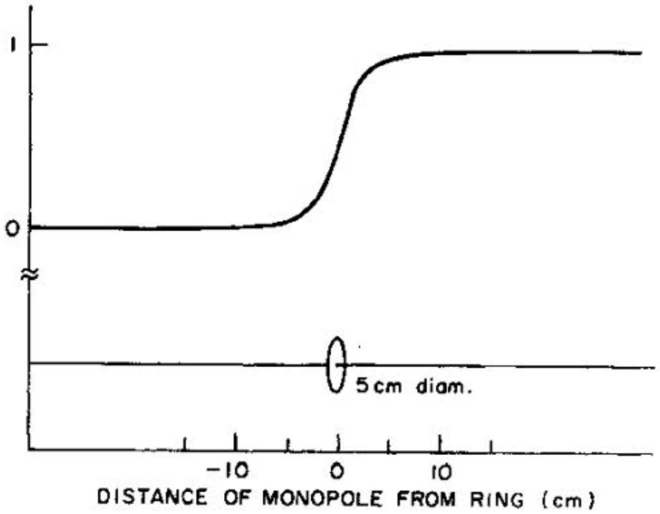
Blas Cabrera's experiment

- Monopoles were never seen on earth, but is possible to look for them in cosmic radiation

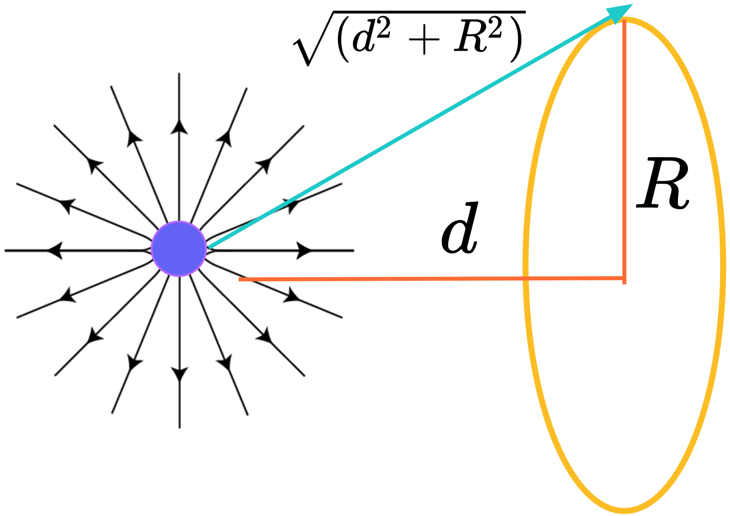
- If a monopole passes through a superconductive coil, it will produce a current

$$\nabla \times \vec{E} = -\frac{1}{c}\frac{\partial \vec{B}}{\partial t} - \frac{4\pi}{c}\vec{j}_m$$

$$\oint \vec{E} \cdot d\vec{l} = -\frac{1}{c}\frac{\partial \Phi_m}{\partial t} - \frac{4\pi}{c}\vec{I}_m$$



Blas Cabrera's experiment



- Taking t=0 as the time d=0:
  - t<0

$$\Phi_B = 2\pi g[1 + \frac{vt}{\sqrt{((vt)^2 + R^2)}}]$$

  - t>0

$$\Phi_B = 2\pi g[-1 + \frac{vt}{\sqrt{((vt)^2 + R^2)}}]$$

$$\Phi_B = \int_A \vec{B} \cdot d\vec{S}$$

$$\Phi_B = 2\pi g[1 - \frac{d}{\sqrt{(d^2 + R^2)}}]$$

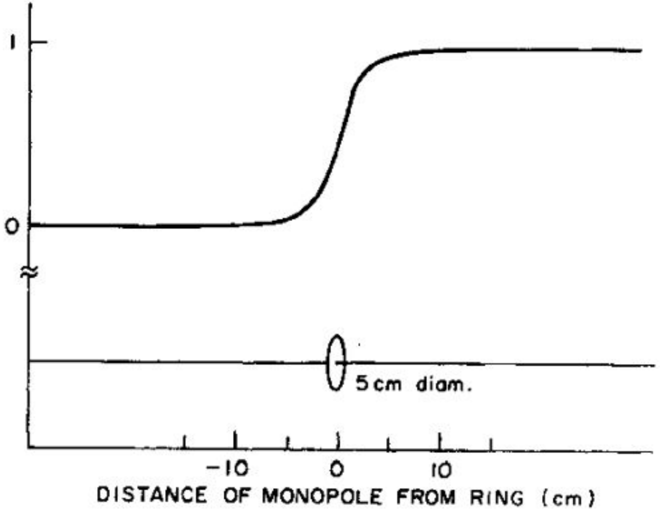
$$\boxed{\Phi_B = 2\pi g[1 - 2\Theta(t) + \frac{vt}{\sqrt{((vt)^2 + R^2)}}]}$$

Blas Cabrera's experiment

$$-LI_e(t) = -\tfrac{1}{c}\Phi_m - \tfrac{4\pi}{c} g\Theta(t)$$

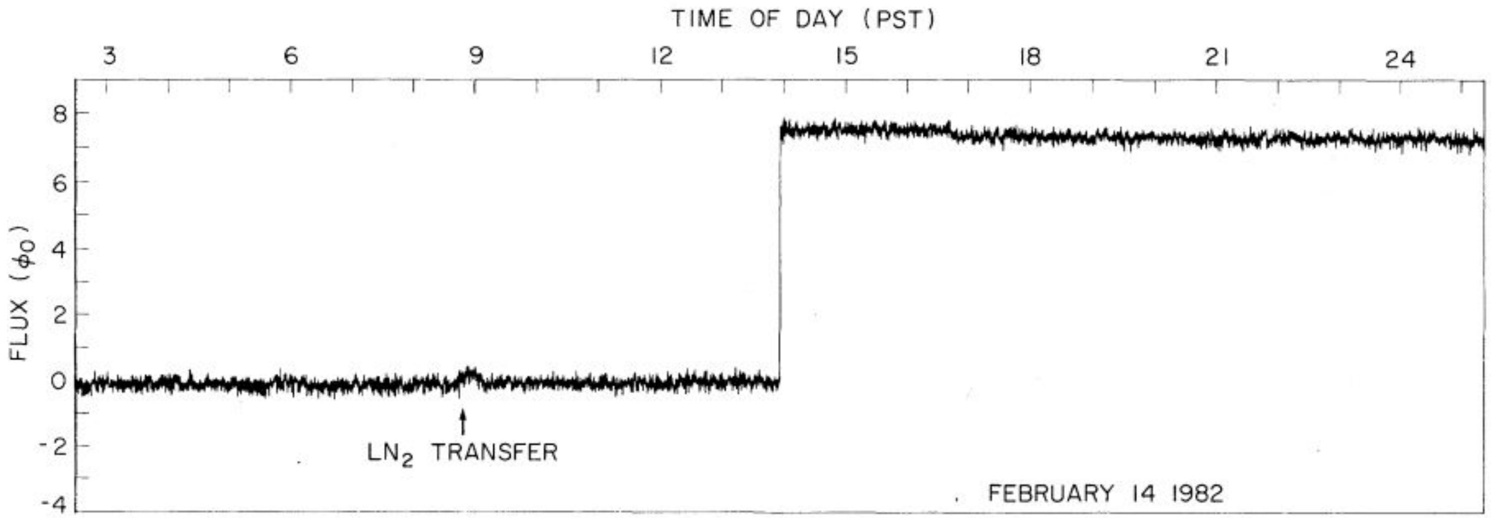
$$\Phi_B = 2\pi g[1 - 2\Theta(t) + \frac{vt}{\sqrt{((vt)^2+R^2)}}]$$

$$I(t) = \frac{2\pi g}{Lc}\left(1 + \frac{vt}{\sqrt{(vt)^2+R^2}})\right)$$



$$\boxed{\begin{aligned} &\lim_{t\to+\infty} I(t) = \frac{4\pi g}{L} \\ &\lim_{t\to-\infty} I(t) = 0 \end{aligned}}$$

Blas Cabrera's experiment



Other experiments

Monopole, Astrophysics and Cosmic Ray Observatory (MACRO)

- MACRO operated from 1989 to 2002 with a detection area of 10000m^2, never detecting one.
- No one ever saw any evidence of monopoles in space after 1982!
- If monopoles exist, would we be able to find them?

The poynting theorem with magnetic current

Poynting vector:

$$\vec{S} = \frac{c}{4\pi}\vec{E} \times \vec{B}$$

$$\nabla \cdot \vec{S} = -\frac{1}{4\pi}[\vec{B} \cdot \frac{\partial \vec{B}}{\partial t} + \vec{E} \cdot \frac{\partial \vec{E}}{\partial t}] - \vec{B} \cdot \vec{j}_m - \vec{E} \cdot \vec{j}_e$$

Field's energy density:

$$u = \frac{E^2 + B^2}{8\pi}$$

$$\frac{\partial u}{\partial t} = \frac{1}{4\pi}[E\frac{\partial E}{\partial t} + B\frac{\partial B}{\partial t}]$$

$$\nabla \cdot \vec{S} + \frac{\partial u}{\partial t} = -\vec{B} \cdot \vec{j}_m - \vec{E} \cdot \vec{j}_e$$

The higher the currents, the faster the energy drops

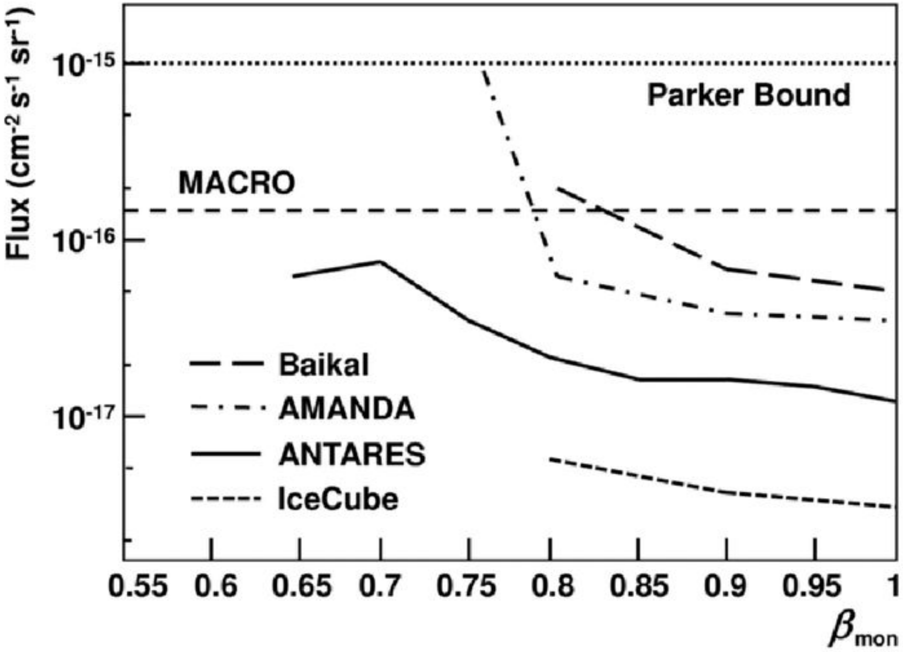
Astrophysical bounds

$$\nabla \cdot \vec{S} + \frac{\partial u}{\partial t} = -\vec{B} \cdot \vec{j}_m - \vec{E} \cdot \vec{j}_e$$

As the galaxy has a longing magnetic field of $3\mu G$ this drop cannot be so fast around here, so there is a limit to the magnetic current.

The flux F of magnetic field can be estimated. By Parker's bound, it would be

$$F \leq 10^{-15}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}\,\mathrm{sr}^{-1}$$



Astrophysical bounds

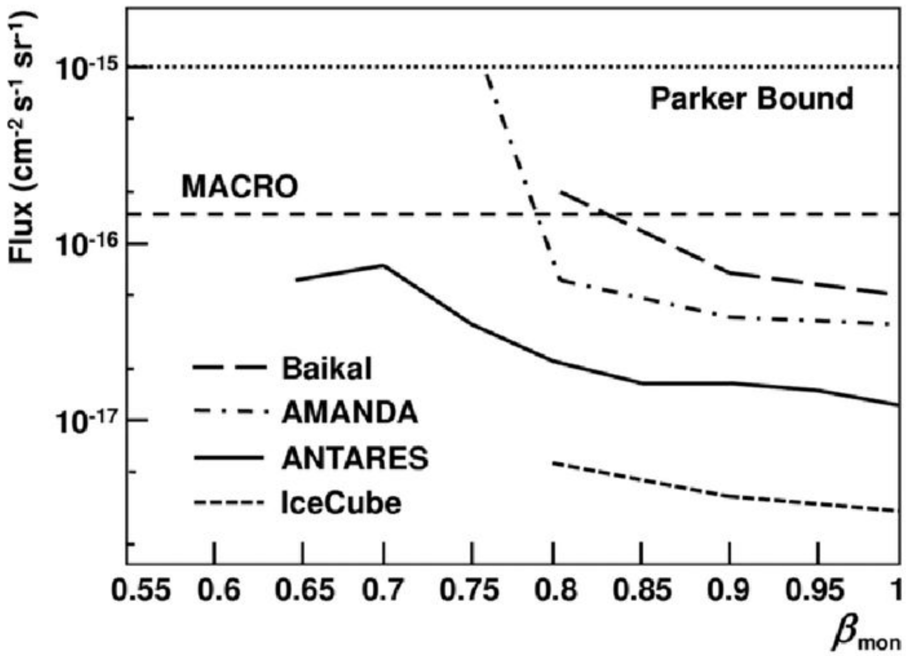
$$F \leq 10^{-15}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}\,\mathrm{sr}^{-1}$$

Considering a detector the size of MACRO (10000m^2), the rate of passing monopoles would be.

$$\frac{N}{t} \leq 1.5 \cdot 10^{-18}\,s^{-1}$$

$$\frac{N}{t} \leq 5 \cdot 10^{-3}\,billion\ years^{-1}$$

One every 20 billion years!?



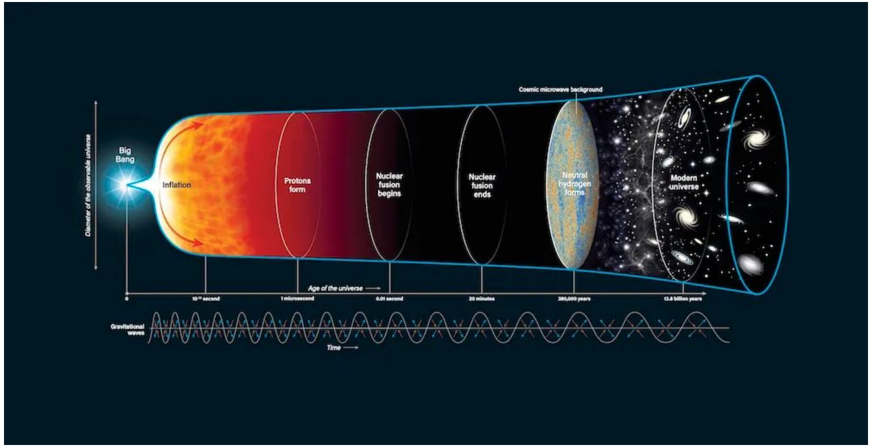
Astrophysical bounds

- If they were as massive as expected ($E \sim 10^{15}$ GeV), they would have formed in the Big Bang.

- To compensate for their gravitational attraction, we need cosmic inflation! ("The monopole problem")

- If that's true, we will never find a monopole.

- Can we make one?



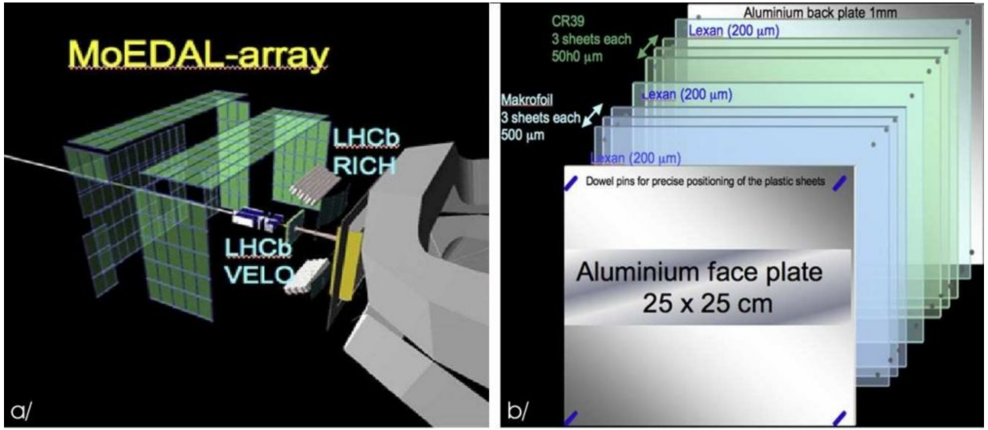
3ST

Creating a magnetic monopole

MoEDAL

- Although not predicted, Intermediate Mass Monopoles are still allowed.

- The Monopole and Exotics Detector at the LHC(MoEDAL) , with collision energies of 8TeV, has been trying to create a monopole since 2009.



4ST

Conclusion

Conclusion

- Magnetic monopoles are allowed under ED and QM
- Gauge field theories expect them at higher energies
- We never find one, and probably never will...
- MoEDAL is the most modern experiment currently trying to prove their existence

Nobel prizes

Paul Dirac(1933)
Antimatter prediction

Carl Anderson(1936)
Discovery of the positron

References

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Thank you!

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